# Representing Words with Vectors 

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## Agenda for the day

- Learning Word Representations
- GloVe model
- Skip-Grams
- CBOW
- FastText


## Overview

# (1) Learning Word Representations 

(2) GloVe model
(3) Skip-gram Model

4 CBOW model
(5) FastText

## Word Representations

- Number of words in human language are far too numerous
- One-hot encoding doesn't capture relationships between words
- Compact representations would make the math work easier / training models easier
- Would be useful to capture Synonyms / Homonyms / Antonyms in these representations
- Would be useful to capture other relationships (e.g. King:Queen :: Man:Woman)


## Word Representations: Some Assumptions

- Words that appear in similar contexts have similar meaning
- Co-occurrence of words convey meaning / structure of language
- Sub-word structures exist in languages


## Word Representations

## Goal

If we convert words into vectors in such a way that words with similar meanings will have vectors that lie nearby; Further if we can do vector arithmetic on them, it would be great. E.g. King - Man + Woman = Queen

## Word Representations

## Representing Words by Vectors

We want to do something like: King $\rightarrow(0.3,0.9,0.9,0.2)$, Queen $\rightarrow(0.3,0.9,0.1,0.2)$ etc

Word Representations


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## GloVe: Global Vectors for Word Representations

| Pr. and Ratio | $\mathrm{k}=$ solid | $\mathrm{k}=$ gas | $\mathrm{k}=$ water | $\mathrm{k}=$ fashion |
| :---: | :---: | :---: | :---: | :---: |
| $P(k \mid$ ice $)$ | $1.9 E-4$ | $6.6 E-5$ | $3.0 E-3$ | $1.7 E-5$ |
| $P(k \mid$ steam $)$ | $2.2 E-5$ | $7.8 E-4$ | $2.2 E-3$ | $1.8 E-5$ |
| $P(k \mid$ ice $) / P(k \mid$ steam $)$ | 8.9 | $8.5 E-2$ | 1.36 | 0.96 |

Table: Co-occurrence Probabilities and their ratios from a 6 Billion word corpus

## GloVe model motivation

- Perhaps model a pair of words and their context as $F\left(w_{i}, w_{j}, \tilde{w}_{k}\right)=\frac{P_{i k}}{P_{j k}}$, where $w$ is the vector representation we desire


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$F\left(\left(w_{i}-w_{j}\right)^{T} \tilde{w}_{k}\right)=\frac{P_{i k}}{P_{j k}}$
- The distinction between $w$ and $\tilde{w}$ is arbitrary. Applying

Homomorphism: $F\left(\left(w_{i}-w_{j}\right)^{T} \tilde{w}_{k}\right)=\frac{F\left(w_{i}^{T} \tilde{w_{k}}\right)}{F\left(w_{j}^{T} \tilde{w_{k}}\right)}$

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- Introduce a weighting function $f\left(X_{i k}\right)$, giving us a new loss function to minimize:
- $J=\sum_{i, k}^{V} f\left(X_{i k}\right)\left(w_{i}^{T} \tilde{w}_{k}+b_{i}+\tilde{b}_{k}-\log \left(X_{i k}\right)\right)^{2}$



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## Skip-gram Model

## Predict a context word, given an input word

Given brown, what is the probability that the, quick, fox, jumps appear in its neighborhood in a sentence


## Source Text

| The | quick | brown |
| :--- | :--- | :--- |
| fox jumps over the lazy dog. | $\Longrightarrow$ |  |


| The | quick | brown | fox |
| :--- | :--- | :--- | :--- |
| jumps over the lazy dog. | $\Longrightarrow$ |  |  |


| The | quick | brown | fox | jumps |
| :--- | :--- | :--- | :--- | :--- |
| over the | lazy dog. $\Rightarrow$ |  |  |  |


(brown, the)
(brown, quick) (brown, fox) (brown, jumps)

## Training

 Samples(the, quick)
(the, brown)
(quick, the) (quick, brown) (quick, fox)
(bus)
(fox, quick) (fox, brown) (fox, jumps) (fox, over)

## Skip-gram Model Training

- $P\left(w_{c} \mid w_{t}\right)=\frac{\exp ^{s\left(w_{t}, w_{c}\right)}}{\sum_{j=1}^{V} \exp ^{s\left(w_{t}, w_{j}\right)}}$
- Online training, using SGD


## Skip-gram Model Training

- Consider special n-grams as single words: New York, Boston Globe
- Negative sampling to selectively at random update a few negative samples. Frequent words have a higher chance of being selected for negative sampling
- Sub-sample frequent words


## Graph for (sqrt(x/0.001)+1)*0.001/x



Figure: Plot of word frequency and Probability of keeping. Empirically obtained

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## CBOW Model

- Instead of predicting the context, predict the target given the context
- Given (the, quick, fox, jumps), predict brown

INPUT PRONECTION OUTPUT


## CBOW Model

- Works better than Skip-gram when corpus is smaller
- Embeddings are averaged across the context, perhaps resulting in more stable representations



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- They all ignore sub-word structures
- Many languages have distinct structures for words
- Many word forms occur rarely even in large corpora, preventing learning good representations for them


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- E.g. Where is modeled as (<wh,whe,her,ere,re $>,<$ where $>$ )
- $s(w, c)=\sum_{g \in G_{w}} z_{g}^{T} v_{c}$
- $P(c \mid w)=\frac{\exp ^{s(w, c)}}{\sum_{j=1}^{V} \exp ^{s(w, j)}}$


