### Feature Engineering, Model Evaluations

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Feature Engineering

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### Overview

### 1) ETL

- Feature Engineering
  - Handling Numerical Features
  - Handling Categorical Features
- Oealing with correlated features
  - Feature Selection
    - Forward and Backward Selection
    - Via Regularization
- 5 Evaluating Model Performance
  - Handling rare classes

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### ETL: Preparing your data for modeling

- Frequently, the data that you collect is in long format
- For example, logging systems will put timestamps, some entity id and then a few columns
- In addition, you might need to join multiple inputs together
- Further, you may want to apply some transformations / aggregations to the individual columns
- All of these can be considered jointly as an **ETL** operation
- Frequently in Data Science, we end up with **ELT**. That is, the Transformation step is done closer to modeling stage

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# ETL: Preparing your data for modeling

|     | Month | Day | Feature | Value |
|-----|-------|-----|---------|-------|
| 1   | 5     | 1   | Ozone   | 41    |
| 2   | 5     | 2   | Ozone   | 36    |
| 3   | 5     | 5   | Ozone   | NA    |
|     |       |     |         |       |
| 610 | 9     | 28  | Temp    | 75    |
| 611 | 9     | 29  | Temp    | 76    |
| 612 | 9     | 30  | Temp    | 68    |

#### Pivot

Typical output found in a log file. You need to convert this into usable formats by pivoting the individual rows

Month Day Ozone Solar.R Wind Temp 5 1 41 190 7.4 67

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- In addition to pivoting, you might look at various aggregations / combinations
- How many times did this event occur
- In the past time window at this location how many times did the event occur
- How much away from the mean does this event deviate
- Usually task dependent with deep subject matter expertise, you ask more meaningful questions

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- Count How many times did something happen
- Amount How much was it
- Continuous Both positive and negative values have meaning

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- Standardize the numerical variables
- Min-max normalization
- sigmoid / tanh / log transformations
- Handling zeros distinctly potentially important for Count based features
- Decorrelate / transform variables

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# One Hot Encoding

- Transforms an attribute taking d distinct values into a d - 1 dimensional binary vector
- Simple, Easy and gives a great deal of interpretability of the attribute
- Great technique for small number of distinct values
- Categorical attribute with large number of distinct values becomes an issue (e.g. zipcode/MSA/items)

| Label |   |   |   |
|-------|---|---|---|
| Dog   | 1 | 0 | 0 |
| Cat   | 0 | 1 | 0 |
| Mouse | 0 | 0 | 1 |

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### Target Rate Encoding

- Compute  $P(c|\mathbf{d})$  empirically
- Replace attribute by a k - 1-dimensional vector (one for each target class)
- Statistics need to be robust enough (take care of distinct values with small counts)
- Handling missing values those not in training partition

| Feature | Target |
|---------|--------|
| Up      | 1      |
| Up      | 0      |
| Down    | 1      |
| Flat    | 0      |
| Flat    | 1      |
| Down    | 0      |
| Up      | 1      |
| Down    | 0      |

| Feature | Encoding |  |
|---------|----------|--|
| Up      | 0.66     |  |
| Down    | 0.33     |  |
| Flat    | 0.5      |  |

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- Use summary statistics to reduce the number of distinct values
- Use Informative measures such as TF-IDF (https://en.wikipedia.org/wiki/TfĐidf)
  - TF and IDF can both be computed in several ways
  - TF: raw, boolean, log-scaled, augmented (to account of document size variations)
  - IDF: raw, log-scaled, smoothed log-scaled, ...
- Entropy  $(-p_i \log p_i)$  to select distinct values of importance

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- low-rank approximation is an optimization technique
- **Cost** is the closeness of fit between original matrix and **low-rank** approximation
- PCA/SVD
- Non-negative Matrix Factorization (NMF)  $\Theta^T X \approx Y$ . Use Gradient Descent, for instance

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# LSH/Random Hashing

- When dimensionality is really large, often times a random low-dim projection seems to work
- That is, take your really large dimensional vector and project it down to a really small space randomly
- Collisions will occur but they don't seem to matter. Interpretability is lost, of course
- LSH (Locality Sensitive Hashing) is a more principled approach that does the same
- Several types of projections (Walsh-Hadamard matrix, random Gaussian matrix) have proven distance preserving properties
- https:

//en.wikipedia.org/wiki/Locality-sensitive\_hashing

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- Useful technique to analyze the inter-relationships between variables
- Feature dimensionality reduction with minimum loss of information
- $y_i = \sum_{j=1}^k \beta_{ij} x_j$
- $y_i = \beta_i^T X$ ,  $Y = \beta^T X$
- $\Sigma$  covariance matrix of X and  $\Sigma_Y$  covariance matrix of Y

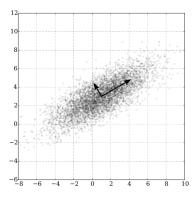
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- $\Sigma_Y = \beta^T \Sigma \beta$ •  $var(y_i) = \sum_{p=1}^k \sum_{m=1}^k \beta_{ip} \beta_{im} \sigma_{pm} = \beta_i^T \Sigma \beta_i$
- $covar(y_i, y_j) = \sum_{p=1}^k \sum_{m=1}^k \beta_{ip} \beta_{jm} \sigma_{pm} = \beta_i^T \Sigma \beta_j$
- Spectral Decomposition Theorem:  $\Sigma=\beta D\beta^T$  , where  $\beta$  contains the eigenvectors
- Easy to show that  $var(y_i) = \lambda_i$  and  $covar(y_i, y_j) = 0$
- $trace(\Sigma) = \sum_{i=1}^{k} \sigma_i^2 = \sum_{i=1}^{k} \lambda_i$
- Total variance of the data is accounted in the eigenvalues. Choosing the top l << k eigenvalues and their corresponding eigenvectors gives you a distortion minimizing low-dimensional projection

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### Linear Discriminant Analysis

- PCA comes up with a projection that represents the data in a low-dimensional feature space
- It is optimized towards **representation**. We are interested in **discrimination**



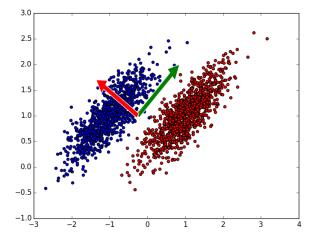
#### Figure: Gaussian Data

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### PCA vs LDA: An Example



#### Figure: PCA vs LDA comparison

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### LDA

- Motivated from Bayes' Rule:  $P(c|\mathbf{x}) = \frac{P(\mathbf{x}|c)P(c)}{\sum_d P(\mathbf{x}|d)P(d)}$
- Assume Gaussian distributed data
- Further assume that, classes have identical covariance
- End up with:  $f_i=\mu_i\Sigma^{-1}x^T-\frac{1}{2}\mu_i\Sigma^{-1}\mu_i^T+ln(P(i))$  which is a line
- See <a href="http://people.revoledu.com/kardi/tutorial/LDA/">http://people.revoledu.com/kardi/tutorial/LDA/</a> for a nice tutorial
- See http://www.ics.uci.edu/%7Ewelling/classnotes/papers\_ class/Fisher-LDA.pdf for a detailed explanation
- The most discriminant projection comes out as eigenvectors of  $\Sigma^{-1}\Sigma_b$

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- Not all features are important
- Often times, removing features from the model improves its performance
- If there are N features, there are  $2^N$  subsets  $\implies$  feature selection is a hard task!

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- Build a model with one feature
  - From features, build models in turn and retain the model with the strongest performance
- From the remaining features, keep adding one feature at a time (using search method above)
- Stop if there is no significant improvement

- Similar to Forward Selection Procedure
- Build model with all features
- Remove the **weakest** feature
- Keep removing as long as there is no significant change

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- The more parameters a model has, the more flexible it is
- Too much flexibility leads to overfitting. Model learns the peculiarities of a particular data set and doesn't generalize well
- Solution: Simplify the model
- Everything must be made as simple as possible, but not simpler!
- We can add a penalty term to the objective function and do joint minimization
- $\bullet \underset{\Theta}{argminJ(\Theta)} + \lambda ||\Theta||$

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# $\mathbf{L}_2$ Regularization / Ridge Regression

•  $\underset{\Theta}{\operatorname{argmin}J(\Theta)} + \lambda \sum_{i} \theta_{i}^{2}$ 

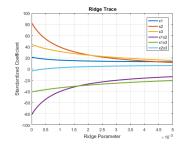


Figure:  $\theta$  values as a function of  $\lambda$ 

In the end, you get regularized weights of features. Some weights are 0 or close to 0. These features are not important to the model.

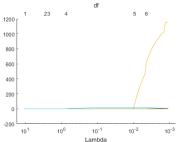
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### $L_1$ Regularization / LASSO

```
• argminJ(\Theta) + \lambda \sum_{i} |\theta_{i}|
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Trace Plot of coefficients fit by Lasso

Figure:  $\theta$  values as a function of  $\lambda$ 

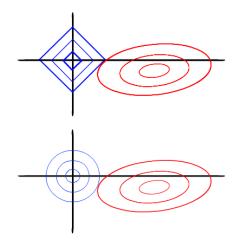
http://statweb.stanford.edu/~owen/courses/305/ Rudyregularization.pdf

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### Geometric View of Lasso and Ridge



#### Figure: Lasso and Ridge Regularization

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- If there are a set of highly correlated features, Lasso ends up selecting only one of them
- When we have high dimensionality, low sample count problem, Lasso saturates quickly
- Ridge doesn't achieve the sparsity achieved by Lasso
- $\underset{\Theta}{\operatorname{argminJ}}(\Theta) + \lambda_1 \sum_i |\theta_i| + \lambda_2 \sum_i \theta_i^2$
- Combines the advantages of both Lasso and Ridge, especially for large dimensionality problems

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- Accuracy.  $\frac{\#Correct}{\#Samples}$
- Confusion matrix

$$\begin{array}{|c|c|c|} Pred. + ve & Pred. - ve \\ Actual + ve & TP & FN \\ Actual - ve & FP & TN \end{array}$$

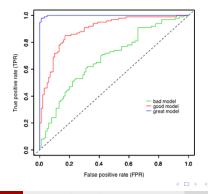
• Precision:  $\frac{TP}{TP+FP}$ , Recall:  $\frac{TP}{TP+FN}$ 

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### Receiver Operating Characteristic

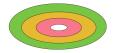
- Precision and Recall reflect a particular setting of decision thresholds
- E.g. Logistic Regression Score / Naive Bayes Neg. Log Likelihood select any threshold you want
- ROC curve shows the performance for any possible choice of this threshold. AUC – Normalized under ROC curve is another number you can use to compare models



- Often, the class of interest is very rare
- E.g. detecting a rare disease
- Identifying a purchaser from the average internet browser
- Default answer of **No** has a very high accuracy
- But, that is not a very useful classifier

### Handling rare class classification

- Penalize different classes differently
- Subsample the background class / Up-sample the foreground class
- Construct hierarchical / back-off classifiers (Russian doll model)
  - Browser / Site Visitor
  - Shopping Cart / Site Visitor
  - Purchase / Shopping Cart



#### Figure: Russian Doll Model

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